zircon begins between 1525 and 1550° C, the degree of dissociation remaining small and nearly constant up to ~1650° C, after which it increases rapidly. The results obtained are in excellent agreement with those presented by Curtis and Sowman [1] and, therefore, as far as our studies are concerned, the phase diagram depicted in Fig. 1 appears more reliable that that of Fig. 2.

It must be pointed out that natual zircon constitutes the basis of many refractory materials and a knowledge of the parameters of solid state dissociation of zircon is of importance especially in order to avoid the presence of free silica, which lessens the chemical and mechanical properties of such materials.

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## *Poisson contraction in aligned fibre composites showing pull-out*

When an aligned fibre composite is stretched parallel to the fibres tractions arise across the interface due to the difference in Poisson's ratio between fibre and matrix. This Poisson contraction is usually simple to deal with in the aligned case, because fibres and matrix are subject to the same longitudinal strain. However, in a number of experiments, e.g. pull-out of single fibres and other methods designed to measure the interfacial shear strength, the longitudinal strains in the two components are not the same. In such cases, somewhat different results may be obtained depending upon whether a single fibre or an array of fibres is considered. This is of particular importance in considering interfacial tractions in composites undergoing multiple fracture. In this note we wish to point out why this is so, and to state succinctly some of the consequences.

Consider first a single continuous fibre with Poisson's ratio $\nu_f$ embedded in matrix with Poisson's ratio $\nu_m$ (Fig. 1), and suppose the whole composite is stretched by an axial strain $e$. When both fibre and matrix are isotropic, the elastic problem is one of circular symmetry, and is easily solved exactly [1, 2]. The interfacial traction is a normal stress $p$ given by

$$p = \frac{2e(\nu_m - \nu_f)V_m}{\left[\frac{V_m}{k_f} + \frac{V_f}{k_m} + \frac{1}{G_m}\right]} \quad (1)$$

where $k$ is the plane strain bulk modulus and $G$

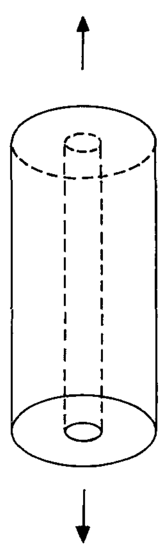

*Figure 1* Single fibre imbedded in a matrix.

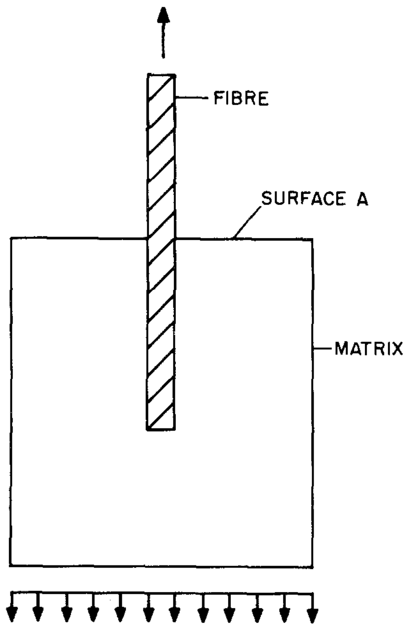


*Figure 2* Fibre pull-out test.

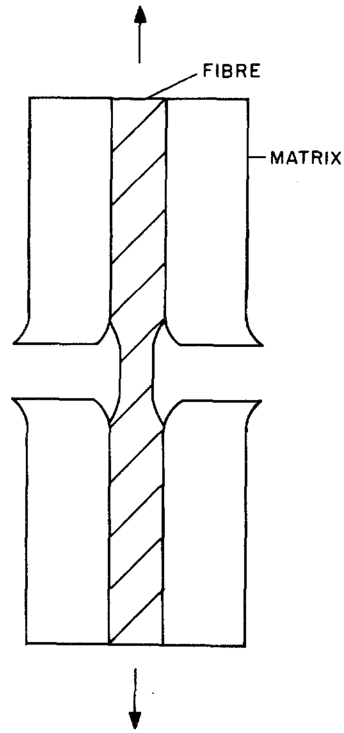


*Figure 3* Representation of fibre and matrix in the vicinity of a crack. The Poisson contractions have been grossly exaggerated for clarity.

the shear modulus. The subscripts f and m refer to fibre and matrix. $V$ is the volume fraction. The sign convention is such that $p$ is positive if the central fibre is under compression.

The sign of the interfacial traction depends on $(\nu_m - \nu_f)$, since the shear modulus and plane strain bulk modulus must be positive, and hence is compressive for the case when the fibre Poissons ratio is less than that of the matrix. This is usually the case with advanced composites. Correpondingly, if the composite is in compression the interfacial traction will be tensile, tending to separate the materials. This is the case in the method described by Outwater and Murphy for measuring the interfacial shear stress [3], where the whole composite is subject to compression.

In the fibre pull-out test, as described by e.g. [4], interfacial traction is likely to be tensile also (see Fig. 2). This occurs because the tensile strain in the fibre is larger than that in the matrix. In the matrix, the average strain at the surface A in Fig. 2 is clearly zero, whereas in the fibre it is finite. The fibre–matrix interface is, therefore, under tension close to the surface independently of the values of the Poisson's ratios of the fibre and matrix.

In both of these methods of attempting to measure the interfacial shear strength there is the possibility, therefore (in the absence of residual stresses), of an unstable debonding failure at the interface. This unstable debond is likely to make measurements of interface properties such as the interfacial sliding friction stress unreproducible and it is known that the values of the shear stress opposing relative motion of fibre and matrix are not very consistent. They may be consistent if the method of making the specimen results in the contraction of the matrix so that there is a normal pressure $p$ across the interface and this is sufficiently large to prevent the unstable debonding due to the Poisson effect. We are of the opinion that when such measurements do yield consistent results that contraction of the matrix must have occurred prior to the pull-out test. The contraction can be due to shrinkage on curing or on change of temperature or due to other causes. These may all be included in the statement that residual stresses must be present which place the interface under a normal pressure.

In the absence of residual stresses the interface is unstable in a single fibre pull-out test. However, this may not be so in a test where a number of fibres are pulled out simultaneously. This case is important since the geometry and relative stresses involved are similar to those occurring in experiments on multiple fracture of brittle matrix composites.

The situation we envisage in this case is shown in Fig. 3 where the matrix in Fig. 1 has cracked on a single plane across the specimen. Far from the crack, the strain in both fibre and matrix have the same value, $e$. At the crack the fibre has undergone an additional strain $\alpha e$ where $\alpha = (E_m V_m)/(E_f V_f)$ and an additional Poisson contraction $\nu_f \alpha e$. The matrix, on the other hand, has zero axial strain at the crack face and hence has a strain difference of $-e$ with respect to that of both the fibre and the matrix remote from the crack. The corresponding difference in Poisson contraction with respect to the unstrained matrix is $-\nu_m e$.

The interfacial traction is clearly tensile at the crack, whereas remote from the crack it is compressive (at least for the case when $\nu_f < \nu_m$). Note that the external shape of the specimen has been altered. This change in shape is important because if there is an array of fibres, the lateral expansion of the matrix on shedding load is restrained by the fibres, and hence, despite the local tensile strains at the interface, the fibres and matrix may still be held in contact.

If the fibre and matrix are held in contact then a frictional force must be developed between them on any further extension of the fibres. Multiple fractures can then develop as has been described for many systems, e.g. glass fibres in cement [5], steel wires in epoxy resin [6], carbon fibre in cement [7]. If they are not, the specimen will break with a single crack transverse to the fibres and the fibres will pull out of the matrix, whatever their length.

To consider this case quantitatively, see Fig. 4. In order to assure contact, the expansion of the matrix must offset fibre contraction caused by the increased stress in the fibres. If the centre-to-centre spacing of two fibres far from the crack is $R$, and the composite Poisson's ratio is $\nu_c$, then the condition for contact between fibres and matrix to be maintained after cracking is

$$(R - 2r)\nu_c e - 2r\alpha\nu_f e \geqslant 0 \qquad (2)$$

or, writing $V_f = \pi r^2/R^2$ (which is equivalent to assuming that the fibres are arranged in a square

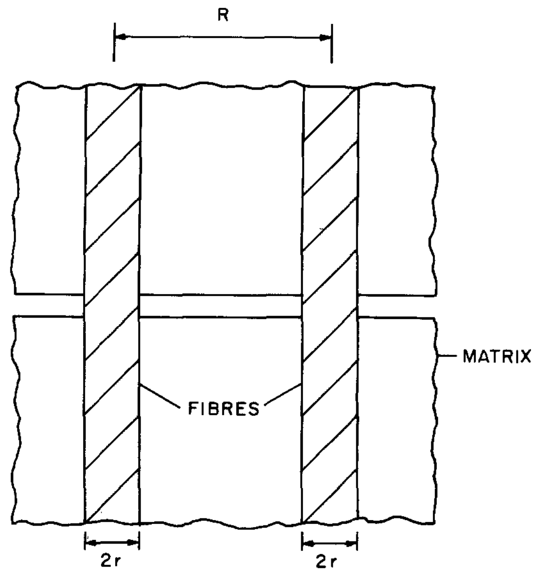


*Figure 4* Cracked surface in a composite reinforced with a parallel array of fibres.

array), we obtain the equivalent condition

$$(\pi^{1/2} - 2V_f^{1/2})\nu_c - 2V_f^{1/2}\alpha\nu_f \geqslant 0. \qquad (3)$$

For a hexagonal array where we take $(r^2/R^2)(2\pi/\sqrt{3}) = V_f$ we would have

$$\left[\left(\frac{2\pi}{\sqrt{3}}\right)^{1/2} - 2V_f^{1/2}\right]\nu_c - 2V_f^{1/2}\alpha\nu_f \geqslant 0. \quad (3a)$$

If these conditions are not obeyed then in the absence of residual stresses in the matrix (e.g. due to curing of a polymer resin or cementation or due to differential thermal contraction between fibres and matrix) multiple fracture will not occur. We wish to explore whether or not these conditions are obeyed in order to find out which systems can show multiple fracture in the absence of any residual stress.

Since the difference between Equations 3 and 3a is not large we shall use 3 in what follows, referring to 3a only when necessary. To explore the variation with volume fraction, Equation 3 may be rewritten substituting for $\alpha$, as

$$\frac{E_f\nu_c}{E_m\nu_f} - \frac{2(1 - V_f)}{(\pi V_f)^{1/2} - 2V_f} \geqslant 0. \qquad (4)$$

For simplicity, we temporarily neglect the variation of composite Poisson's ratio $\nu_c$ with fibre volume fraction. This can be justified because of

TABLE I

| System fibre/matrix | $V_f$ | $E_f$ (GN m$^{-2}$) | $\nu_f$ | $E_m$ (GN m$^{-2}$) | $\nu_m$ | $\nu_c^*$ | $\frac{E_f \nu_c}{E_m \nu_f}$ | $F_s(V_f)$ | $F_h(V_f)$ | Equation 3 satisfied | Equation 3a satisfied |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Steel/cement | 0.05 | 200 | 0.28 | 20 | 0.23 | 0.23 | 8.2 | 6.4 | 5.83 | Yes | Yes |
| Steel/epoxy | 0.2 | 200 | 0.28 | 3.5 | 0.3 | 0.30 | 61 | 4.1 | 3.54 | Yes | Yes |
| Graphite/cement | 0.02 | 200 | 0.35 | 20 | 0.23 | 0.23 | 6.6 | 9.3 | 8.54 | No | No |
| Graphite/cement | 0.05 | 200 | 0.35 | 20 | 0.23 | 0.24 | 6.9 | 6.4 | 5.83 | Yes | Yes |
| Glass/epoxy | 0.6 | 72 | 0.25 | 3.5 | 0.3 | 0.27 | 22 | 4.6 | 2.91 | Yes | Yes |
| Graphite/glass | 0.5 | 200 | 0.35 | 72 | 0.25 | 0.30 | 2.4 | 4.0 | 2.88 | No | No |
| Glass/cement | 0.05 | 72 | 0.25 | 20 | 0.23 | 0.23 | 3.3 | 6.4 | 5.83 | No | No |
| Polypropylene/cement | 0.1 | 10 | 0.3 | 20 | 0.23 | 0.24 | 0.40 | 5.0 | 4.47 | No | No |

* $\nu_c$ is usually found to be close to the value given by the equation $\nu_c = \nu_f V_f + \nu_m V_m$ for an aligned composite.

the much greater variation with $V_f$ of the second term which we call $F_s(V_f)$. Since $F_s(V_f)$ increases without bound as $V_f$ approaches zero, it is clear that at very small volume fractions Equation 4 is never obeyed in any system. As $V_f$ increases it may or may not be. The minimum value of $F_s(V_f)$ occurs when $V_f = 0.37$ when it takes on the value 3.73. Therefore, Equation 4 can never be obeyed in any system unless

$$\frac{E_f \nu_c}{E_m \nu_f} > 3.73. \tag{5}$$

For the case of a hexagonal array, $F_s(V_f)$ is replaced by

$$F_h(V_f) = \frac{2(1 - V_f)}{\left(\frac{2\pi V_f}{\sqrt{3}}\right)^{1/2} - 2V_f}.$$

Data for a number of systems are collected in Table I. The first four systems obey criterion 5 showing that unstable debonding of the interface need not occur in aligned composites provided the volume fraction of fibres is suitably chosen. These four systems have also been examined to see whether they are consistent with Equation 4 at the specific volume fractions shown. All of them are consistent with Equation 4 at these volume loadings of fibre except for graphite/cement at $V_f = 0.02$ and in each multiple fracture is known to occur [6, 7, 9]. We conclude that in these systems, with the exception of the case cited, the occurrence of matrix shrinkage is not a necessary condition for the observance of multiple fracture at these volume loadings. We observe from the Table that in graphite/cement at $V_f = 0.02$ a small amount of matrix shrinkage may be required for multiple fracture to occur.

The last entry in Table I is for polypropylene fibre in cement. Criterion 5 is not obeyed. We conclude that multiple fracture will not be observed in cement containing aligned polypropylene fibres at any volume loading since the fibre matrix interface will debond in an unstable fashion. Multiple fracture could only be observed if substantial shrinkage of the cement occurs so that the fibre is gripped. The reason for this is the small Young's modulus of the fibre and its large Poisson's ratio.

The case of glass/cement – the penultimate entry in the Table, is very interesting. Multiple fracture is known to occur [5]. The criteria are not obeyed, which indicates, if the present ideas are correct, that the interfaces between fibre and matrix would debond unstably. However, they are very close to being obeyed and our theory is obviously (quantitatively) approximate. We have assumed a regular square array of fibres. For a hexagonal array the condition analogous to criterion 5 yields

$$\frac{E_f \nu_c}{E_m \nu_f} > 2.88 \qquad (6)$$

instead of 3.73. The value for glass/cement in Table I is 3.3. Further, for the particular volume fraction of 5%, which is a technically interesting one, Equation 3 is close to being obeyed and the value of Poisson's ratio for cement is not known with accuracy. We believe these results indicate that in glass-reinforced cement a small amount of shrinkage of the cement will be necessary in order to observed multiple fracture or else that the interface must be capable of sustaining a tensile traction and be tough enough to resist debonding in order that multiple fracture be consistently observed. The entry in the Table for graphite fibres in glass indicates that our criterion is not obeyed. We know of no experiments in pure tension on this system but there is some evidence for multiple fracture in bend tests [8].

The previous discussion was limited to consideration of arrays of parallel fibres perpendicular to the crack face. When fibres are randomly oriented in the composite, some fibres will intersect the crack obliquely as in Fig. 5. The mechanisms in this case are more complex. As in the case when fibres are axial, oblique fibres will neck down locally because of the increase in strain, and the matrix will tend to expand laterally when the fibres debond, although the oblique fibres will restrict the displacement. In addition, the axial separation of the crack faces tends to force the fibre against the matrix which helps to maintain contact. For these reasons the presence of residual stresses in the matrix may not be necessary in order to observe multiple fracture. This case was studied by Morton and Groves [10].

In conclusion, we have considered some aspects of Poisson contraction in composites reinforced with aligned continuous fibres which are rigorously straight sided and parallel. We found that the effect of the Poisson contraction can be different for single fibres and arrays of fibres in the same matrix material. That is the important conclusion. Experiments are conducted on arrays of fibres which are not necessarily completely smooth so that relative motion of fibres and matrix may be determined by asperities at the interface. Nevertheless, the effects we predict must be taken into account in any exact theory. The analysis predicts that in the absence of shrinkage stresses, interfacial debonding will probably tend to propagate unstably in glass/cement composites with low fibre volume fractions. Interfaces in steel/cement, graphite/cement and in resin-matrix composites reinforced with high volume fractions of fibres such as glass, and steel, are predicted to be mechanically stable.

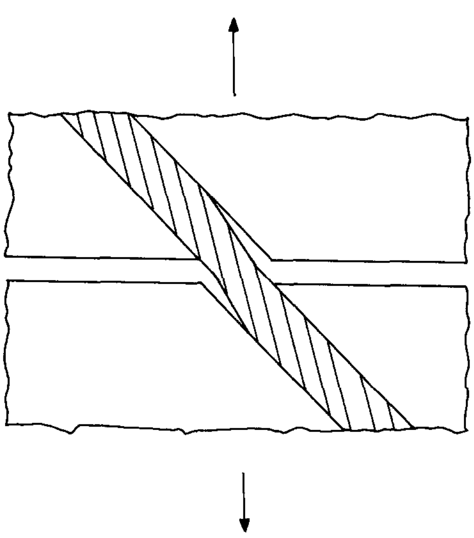

*Figure 5* Fibre intersecting crack at an oblique angle.

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## *Spherical and cylindrical models for craze growth*

In a previous paper [1] we applied the technique of finite element analysis to the problem of failure by yielding of a uniform cylindrical void system, taken as a model for the process of craze growth in plastics. However, the smallest voids formed during the initiation of crazing often seem to have a spherical rather than a cylindrical form [2, 3]. We have, therefore, repeated some of our previous calculations using a spherical hole model to establish whether or not there is a significant difference in the conclusions reached. The new model simply substitutes a three-dimensional system of spherical holes for the two-dimensional cylindrical voids used in the previous paper [1].

A single module of the spherical void array is illustrated in Fig. 1 where the voids are of radius $a$ and their centres are spaced at an equal distance of $2(a + d)$ in each of the $x$, $y$ and $z$ directions. The loading of prime interest is a hydrostatic tensile loading which can be accomplished by prescribing displacements $\Delta$ on the three faces $x = a + d$, $y = a + d$ and $z = a + d$ in the $x$, $y$ and $z$ directions respectively. The three faces defined by $x = 0$, $y = 0$ and $z = 0$ are restrained from moving in the $x$, $y$ and $z$ directions respectively.

Initially, two void volume fractions were considered corresponding to $d/a = 0.5$ and $d/a = 0.1$. In each case the solution was performed using 1, 8 and 27 three-dimensional quadratic elements in turn and the 8 element mesh employed is illustrated in Fig. 1b. A Von Mises yield criterion was assumed and the material properties, listed in Fig. 1a, are the same as those employed in [1]. The variation of the total reactive force on any face with increasing prescribed displacement is shown in Fig. 2a

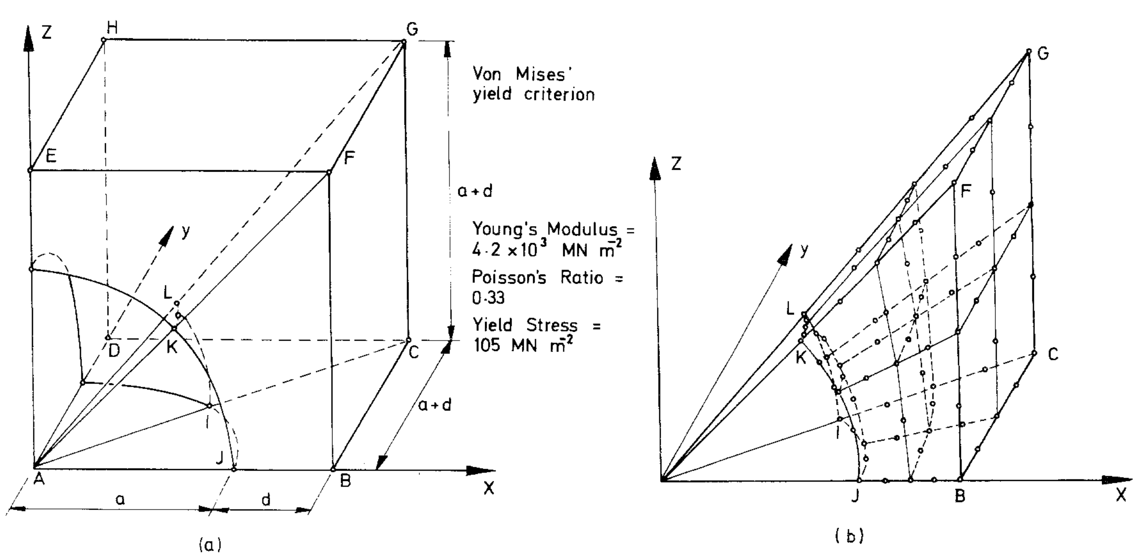


*Figure 1* Illustration of spherical void model. (a) Module analysed; (b) quadratic isoparametric element mesh employed.